Unified Field Theory Based on Quantum Phase Structure of Spacetime: A Geometric Framework from Kaluza–Klein Solitons and Torsion

BTTP*

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Abstract

We propose a geometric framework in which the four fundamental interactions, dark matter, and horizonless alternatives to black holes arise from a single higher-dimensional action with torsion. Gauge fields are realized as isometries of a compact internal space, so that gauge invariance holds as a theorem rather than an assumption; parity violation is encoded geometrically through the interplay of a Holst-type term and non-minimal fermion couplings, giving the parity-nonconservation parameter a geometric origin, $\chi - 1 \propto \alpha/\beta$. We prove a torsion-decoupling lemma showing that the smooth horizonless topological stars of Bah and Heidmann are exact solutions of the full action, thereby embedding their extensively studied phenomenology—shadows, quasinormal-mode spectra, and stability—within a unified geometric setting. We supplement that phenomenology with an analytic treatment of the photon-cavity crossing time, obtaining the closed-form fit $\Delta t_{\text{echo}} \simeq 11.7\,(GM/c^3)\sqrt{(3/2-b)/(b-1)}$ with $b = r_B/r_S$, consistent with the cavity effect found in published quasinormal-mode computations: generic solitons perturb the ringdown spectrum rather than producing well-separated echoes. The shadow-diameter deficit of 33–43% relative to an equal-mass black hole, combined with Event Horizon Telescope measurements, excludes M87* and Sgr A* as solitons of this class while leaving unimaged compact objects as candidates. Smoothness and collider bounds jointly force smooth solitons to be microscopic, yielding a dark-matter candidate near $10^{-23}\,M_\odot$ invisible to microlensing by construction, while exposing a size–hierarchy tension for astrophysical-mass mimickers, which we state explicitly. The framework is presented not as a completed theory of everything but as a falsifiable research program: its relation to prior work, confirmed exclusions, surviving parameter regions, and open problems are catalogued.

1 Introduction

1.1 Background and relation to previous work

Modern physics faces the unified understanding of the four fundamental interactions, the black-hole information paradox, and the nature of dark matter and dark energy. Recently, Lindgren et al. [1] proposed reinterpreting electromagnetism as a purely geometric theory, while Bah and Heidmann [2, 3] constructed smooth horizonless "topological stars" in five-dimensional Einstein–Maxwell theory, whose imaging properties were analyzed in Ref. [4].

The phenomenology of these objects is by now the subject of a substantial literature, which the present framework builds upon rather than duplicates: scalar quasinormal modes and the

*https://www.bttp.info/

associated cavity effect were computed in Ref. [5]; electromagnetic and gravitational perturbations of the dimensionally reduced four-dimensional solution were studied in Refs. [9, 10]; and stability under radial and nonradial perturbations has been analyzed in Refs. [6, 7, 8]. The contribution of this paper is orthogonal to that body of work: we embed the topological-star sector within a candidate unification of all four interactions based on a single torsionful higher-dimensional action, derive the conditions under which the known solutions remain exact solutions of the enlarged theory (the torsion-decoupling lemma of §6), and draw the global consequences—for parity violation, dark matter, and internal consistency—that only become visible at the level of the unified framework.

The present v2.1 also withdraws and corrects all mathematical inconsistencies of v1: a gauge-noninvariant extended metric, a metric ansatz mixing differential forms of different rank, and an erroneous inequality concerning time reversal.

1.2 Objectives

(1) Construction of a single action from which gauge invariance holds as a theorem. (2) Geometrization of parity violation via torsion and a Holst-type term. (3) Proof that horizonless topological stars are exact solutions of the full action, with their observational consequences summarized analytically and situated in the existing literature. (4) Confrontation with existing data (EHT, LIGO–Virgo–KAGRA, LHC, microlensing) to identify surviving parameter regions.

1.3 Fundamental assumptions

(i) Physics derives from a single action of $(4+d)$-dimensional geometry with torsion (§2). (ii) Gauge fields are realized as the isometry group of the internal space. (iii) Chirality and stability originate in the topological defect structure of the internal space.

2 Unified Geometric Action

2.1 Fundamental postulate and total action

All field equations derive from a single action on a $(4+d)$-dimensional manifold $\mathcal{M}_{4+d}$ with metric G_{MN} and independent connection carrying torsion $\mathcal{T}^P{}_{MN} = \Gamma^P{}_{[MN]}$:

$$S = \frac{1}{2\kappa_{4+d}^2} \int d^{4+d}X\sqrt{-G}\Big[R(G,\mathcal{T}) + \frac{1}{\beta}\varepsilon^{MNPQ}{}_{R_1\cdots R_d} R_{MNPQ}\,\sigma^{R_1\cdots R_d} - 2\Lambda_{4+d}\Big] + \int d^{4+d}X\sqrt{-G}\,\frac{i}{2}\Big[\bar{\Psi}\Gamma^M \overrightarrow{D}_M\Psi - \bar{\Psi}\overleftarrow{D}_M\Gamma^M\Psi\Big] + (\text{non-minimal terms}), \quad (1)$$

where $R(G,\mathcal{T})$ is the curvature scalar of the torsionful connection, β the Barbero–Immirzi parameter, σ the covariantly constant internal volume form defining the parity-odd (Holst-type) invariant in $4+d$ dimensions, and D_M the spinor covariant derivative of the full connection. No fundamental Yang–Mills term appears: gauge fields are extracted from geometry, not added to it. Dimensional bookkeeping ($\hbar = c = 1$): $[G_{MN}] = 0$, $[R] = M^2$, $[\kappa_{4+d}^2] = M^{-(2+d)}$, $[\Lambda_{4+d}] = M^2$, $[\Psi] = M^{(3+d)/2}$, $[\beta] = [\sigma] = 0$; every term is dimensionless.

2.2 Gauge fields as internal geometry

The v1 ansatz $g_{\mu\nu} = \eta_{\mu\nu} + \kappa_E A_\mu A_\nu + \kappa_S \sum_a G^a_\mu G^a_\nu$ is abandoned: $A_\mu A_\nu$ is not gauge invariant and cannot appear in an observable geometric quantity. It is replaced by the Kaluza–Klein ansatz on

$\mathcal{M}_4 \times \mathcal{K}_d$:

$$ds^2_{4+d} = g_{\mu\nu}(x)dx^\mu dx^\nu + \gamma_{mn}(y)\big[dy^m + K^m_a A^a_\mu dx^\mu\big]\big[dy^n + K^n_b A^b_\nu dx^\nu\big], \tag{2}$$

with γ_{mn} the metric on compact $\mathcal{K}_d$ and K^m_a its Killing vectors, $[K_a, K_b] = f^c{}_{ab}K_c$. Gauge invariance is a theorem: under $y^m \to y^m + \epsilon^a(x)K^m_a(y)$ the ansatz retains its form iff

$$A^a_\mu \to A^a_\mu - \partial_\mu \epsilon^a - f^a{}_{bc}\epsilon^b A^c_\mu, \tag{3}$$

precisely a non-Abelian gauge transformation; every invariant of G_{MN} is exactly gauge invariant. Reduction yields

$$S_{4D} \supset \int d^4x\sqrt{-g}\Big[\frac{R_4}{2\kappa_4^2} - \frac{1}{4g_a^2}F^a_{\mu\nu}F^{a\mu\nu} + \mathcal{L}_{\text{moduli}}\Big], \qquad \frac{1}{g_a^2} = \frac{1}{\kappa^2_{4+d}}\int_{\mathcal{K}_d}\sqrt{\gamma}\,\gamma_{mn}K^m_a K^n_a, \tag{4}$$

so κ_E, κ_S are eliminated: couplings are set by internal geometry, $g_a^2 \sim \kappa_4^2/R_a^2$. Realizing $SU(3) \times SU(2) \times U(1)$ as an isometry requires $d \geq 7$ [11]; we adopt $d = 7$.

2.3 Topological sector and the strong interaction

The second Chern number $\mathcal{C} = \frac{1}{32\pi^2}\int \epsilon^{\mu\nu\rho\sigma}\text{Tr}(F_{\mu\nu}F_{\rho\sigma})d^4x \in \mathbb{Z}$ enters via $S_\theta = \theta\,\mathcal{C}$, descending from higher-dimensional topological terms. Corrections to v1: (i) $\mathcal{C}$ does not by itself produce color confinement, which remains conjectural and is not claimed; (ii) the topological sector quantitatively controls the topological susceptibility and, via the Witten–Veneziano relation [17], the η' mass—the checkable content here.

2.4 Asymptotic freedom

The one-loop running of $g_s^2(Q^2)$ is retained but reinterpreted: since g_a^{-2} is a modulus of $\mathcal{K}_d$, the RG flow is the scale dependence of the moduli effective action. Deriving $(33 - 2n_f)/12\pi$ from it is open (OP8).

3 Fermions, Reduction, and Chirality

3.1 Unified covariant derivative as a theorem

The $(4 + d)$-dimensional Dirac operator, reduced with §2.2, automatically produces $D_\mu = \partial_\mu + \frac{1}{4}\omega_{\mu ab}\gamma^a\gamma^b + A^a_\mu \hat{T}_a +$ (contorsion): unification of spin and gauge connections is a consequence, not a postulate.

3.2 Chiral obstruction and defects

Smooth Riemannian compactifications cannot yield a chiral 4D spectrum [12]. The internal space $\mathcal{K}_7$ therefore carries defects (conical/orbifold loci); chiral zero modes localize there with $n_L - n_R = \text{index}(D_\mathcal{K})$, a topological invariant of the defect configuration. Standard-Model chirality is thus a topological property; the explicit three-generation construction is open (OP5).

4 Geometric Parity Violation

4.1 Withdrawal of the v1 chiral metric

The v1 line element added a rank-2 quadratic form to a rank-3 form and was not a well-defined geometric object; it is withdrawn. Parity violation is encoded in torsion.

4.2 Torsion decomposition and the Holst sector

With $\mathcal{T}_{\lambda\mu\nu} = \frac{2}{3}(g_{\lambda\mu}V_\nu - g_{\lambda\nu}V_\mu) + \frac{1}{6}\epsilon_{\lambda\mu\nu\rho}S^\rho + q_{\lambda\mu\nu}$, minimal coupling alone yields the parity-even contact term $\propto -\kappa^2 J_5 \cdot J_5$. Genuine parity violation arises from the Holst term $(1/\beta)$ with a non-minimal coupling α; integrating out torsion gives [13]

$$\mathcal{L}_{\text{int}} = -\frac{3\kappa_4^2}{16}\frac{\beta^2}{\beta^2+1}\Big[J_5^\mu J_{5\mu} - \frac{2\alpha}{\beta}J_5^\mu J_\mu + \alpha^2 J^\mu J_\mu\Big], \tag{5}$$

whose cross term is parity-odd, giving $\chi - 1 \propto \alpha/\beta$. Qualifications: (i) this yields parity-violating contact terms, not the full $V-A$ structure, which originates in the defect-localized modes of §3.2; (ii) these terms are gravitationally suppressed; the weak coupling strength arises from the $SU(2)$ isometry sector.

4.3 Time direction (corrected)

The v1 inequality under $\tau \to -\tau$ was erroneous: both terms of the autoparallel equation are even under proper-time reversal. Corrected statement: §4.2 interactions violate P but conserve T; by CPT, T violation requires CP violation (CKM phase; potentially θ). The surviving claim: the weak sector is the only observed sector exhibiting CP—hence T—violation. Links to the macroscopic arrow of time are downgraded to conjecture (§8.3).

5 Dark Sector

5.1 Dark matter as solitons of the same action

Since the action admits the horizonless solitons of §6 as classical solutions, dark matter is modeled as a population thereof: $\rho_{DM}(x) = \sum_i n_i(x) M_i$, $M_i = M_{\text{ADM}}[\text{soliton}_i]$.

5.2 Vacuum energy (status clarified)

The topological sum $\rho_\Lambda = l_P^{-4}\sum_{\text{topologies}} e^{-S_E}$ is retained as formal only: the Euclidean gravitational path integral is not convergent as written; no magnitude for Λ is claimed (OP12).

5.3 Dark-energy–dark-matter relation (replaced)

The v1 relation $\rho_{DE} \propto \nabla^2 \rho_{DM}$ contradicts the observed smoothness of dark energy. Replaced by

$$\rho_{DE}(x) = \rho_\Lambda + \frac{\xi}{M_P^2}\nabla_\mu\nabla^\mu \rho_{DM}(x), \qquad |\xi| \ll \frac{\rho_\Lambda M_P^2}{|\nabla^2 \rho_{DM}|_{\text{gal}}}, \tag{6}$$

dimensionally consistent ($[\xi] = 0$) and falsifiable via dark-energy clustering constraints.

6 Topological Solitons as Horizonless Compact Objects

6.1 Embedding and torsion decoupling

Lemma 1 (Torsion decoupling)**.** *On configurations with vanishing spinor fields, the torsion field equation of §2.1 is algebraic and enforces $\mathcal{T} = 0$; on torsion-free configurations the Holst-type term vanishes identically by the first Bianchi identity $\varepsilon^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma}(g) = 0$. For bosonic configurations the field equations reduce exactly to Einstein gravity coupled to the Kaluza–Klein gauge sector.*

Proof sketch. Torsion carries no derivatives; its equation of motion equates contorsion algebraically to the spin density, vanishing for $\Psi = 0$. With $\mathcal{T} = 0$ the connection is Levi-Civita and the parity-odd invariant reduces to $R_{[\mu\nu\rho]\sigma} = 0$. $\square$

This lemma is the precise sense in which the topological stars of Refs. [2, 3]—and, by extension, the phenomenological results of Refs. [4, 5, 6, 7, 8]—are inherited intact by the unified theory of §2. We work in the minimal consistent truncation, 5D Einstein–Maxwell: $S_5 = \int d^5x\sqrt{-g}[R/2\kappa_5^2 - \frac{1}{4}F_{MN}F^{MN}]$. Embedding subtlety, recorded honestly: if F is geometrized as a graviphoton, the reduction generically produces a radion coupling $e^{-a\phi}F^2$, and the solutions hold as written only on stabilized branches; such truncations exist in supergravity [3], the general problem being OP6.

6.2 The exact solution

On $\mathcal{M}_4 \times S^1_y$, following Refs. [2, 3]:

$$ds_5^2 = -f_S dt^2 + f_B dy^2 + \frac{dr^2}{f_S f_B} + r^2 d\Omega_2^2, \quad F = P\sin\theta\, d\theta \wedge d\varphi, \quad f_{S,B} = 1 - \frac{r_{S,B}}{r}. \tag{7}$$

Direct computation gives $R^t{}_t = R^y{}_y = -r_S r_B/2r^4$, and the field equations hold iff

$$P^2 = \frac{3r_S r_B}{2\kappa_5^2}. \tag{8}$$

The $r_S \leftrightarrow r_B$ symmetry exchanges a black-string branch ($r_S > r_B$) with the horizonless branch ($r_B > r_S$).

6.3 Smoothness

For $r_B > r_S$ spacetime terminates at $r = r_B$. With $\rho = 2r_B\sqrt{u/(r_B - r_S)}$, $u = r - r_B$, the (r, y) metric becomes $d\rho^2 + \rho^2\frac{r_B - r_S}{4r_B^3}dy^2$—a smooth origin—iff

$$R_y = \frac{2r_B^{3/2}}{\sqrt{r_B - r_S}}. \tag{9}$$

The geometry is an everywhere-smooth bubble: a cigar capping at $r = r_B$, a round S^2 of area $4\pi r_B^2$ threaded by flux $\oint F = 4\pi P$; no horizon ($f_S > 0$ on the manifold), no curvature singularity ($r = 0$ excised). Dirac quantization $eP = n/2$ then quantizes $r_S r_B$: the defect is topologically protected—the precise sense of the "phase defect" of the title.

6.4 Charges and compactness

With Harmark–Obers conventions [14], $M_{\rm ADM} = \frac{2\pi R_y}{4G_5}(2r_S + r_B)$; a 4D observer with $G_4 = G_5/2\pi R_y$ assigns $r_s^{(4)} = 2G_4 M_{\rm ADM} = r_S + r_B/2$. Equivalently, after reduction and setting $G_4 = 1$, this is $M = (2r_S + r_B)/4$, the normalization used in the topological-star literature [5]; the coefficient convention is therefore consistent.

6.5 Photon dynamics and shadows

Null geodesics with $p_y = 0$ see $V = f_S/r^2$, extremized at $r_{\rm ph} = \frac{3}{2}r_S$, independent of r_B. The convention matches the standard topological-star split. **Class I / first kind** ($\frac{3}{2}r_S < r_B < 2r_S$ on the metastable branch): the only photon shell is the cap shell at $r = r_B$; there is no outer Schwarzschild-like photon sphere and the shadow-ratio formula below does not apply. **Class II / second kind** ($r_S < r_B < \frac{3}{2}r_S$): an inner stable shell lies at $r = r_B$, while an outer unstable photon sphere lies at $\frac{3}{2}r_S$, with $b_{\rm ph} = \frac{3\sqrt{3}}{2}r_S$. This dichotomy coincides with the two regimes whose imaging properties were studied in Ref. [4]; our contribution is the compact expression of its consequence for the shadow diameter against the four-dimensional mass:

$$\frac{b_{\rm ph}^{\rm sol}}{b_{\rm ph}^{\rm Schw}} = \frac{r_S}{r_S + r_B/2} \in \left(\frac{4}{7}, \frac{2}{3}\right), \tag{10}$$

i.e. 33–43% smaller than an equal-mass black hole. With no horizon, the cavity round trip $\Delta t_{\rm echo} = 2\int_{r_B}^{r_{\rm ph}} dr/[f_S\sqrt{f_B}]$ is finite.

6.6 Analytic evaluation of the cavity crossing time

Scalar quasinormal modes of these geometries, including the cavity effect associated with the interior region, were computed numerically in Ref. [5]. The purpose of this subsection is complementary: an analytic estimate of the cavity crossing time in closed form, useful for population-level and order-of-magnitude arguments. With $x = r/r_S$, $b = r_B/r_S \in (1, \frac{3}{2})$: $\Delta t_{\rm echo} = 2r_S I(b)$, $I(b) = \int_b^{3/2} \frac{x^{3/2}dx}{(x-1)\sqrt{x-b}}$. The substitution $x = b + u^2$ smooths the integrand; Simpson quadrature was cross-checked against analytic separation of the arctan core (4-figure agreement). In units of GM/c^3 with $r_S = 2GM/(1 + b/2)$:

b	$I(b)$	$\Delta t_{\rm echo}\,[r_S]$	$\Delta t_{\rm echo}\,[GM/c^3]$
1.05	13.28	26.56	34.8
1.10	9.006	18.01	23.2
1.20	5.723	11.45	14.3
1.30	3.958	7.92	9.60
1.40	2.511	5.02	5.91
1.45	1.703	3.41	3.95

Asymptotics: $\Delta t_{\rm echo} \to \frac{8\pi}{3}\frac{GM}{c^3}(b-1)^{-1/2}$ (power-law, contrasting the logarithmic divergence of Planckian-surface ECO models) and $\Delta t_{\rm echo} \to 16.8\frac{GM}{c^3}\sqrt{3/2-b}$. Fitting formula:

$$\Delta t_{\rm echo} \simeq 11.7\,\frac{GM}{c^3}\sqrt{\frac{3/2-b}{b-1}} \qquad (\lesssim 2\%,\ 1.05 \le b \le 1.45). \tag{11}$$

Scales: 1.7–10 ms for a $60\,M_\odot$ remnant; 1–12 min for Sgr A*. Since $T_{\rm QNM} \simeq 16.8\,GM/c^3$, generic delays (4–35 GM/c^3) are comparable to a single ringdown cycle: generic Class II solitons do *not* produce well-separated echo trains but a modified quasinormal spectrum with spacing $\Delta f \sim 1/\Delta t_{\rm echo}$ ($\approx$ 240 Hz for $60\,M_\odot$ at $b = 1.2$). This conclusion is consistent with, and independently corroborated by, the numerical finding of Ref. [5] that the quasinormal spectrum of single-photon-sphere topological stars closely tracks that of a black hole with the same photon sphere, with cavity-induced modifications. Separated echoes require $b - 1 \lesssim 10^{-3}$. Caveat: this is the eikonal (null-crossing) estimate, with $O(1)$ wave corrections.

6.7 Information and stability

The spacetime is globally hyperbolic with Cauchy surfaces $\mathbb{R}^2 \times S^2$ and no horizon; quantum evolution is unitary in the standard sense—the paradox does not arise rather than being resolved. Limitations: dynamical formation and microstate counting against Bekenstein–Hawking entropy remain open (OP10). Regarding classical stability, the situation is considerably better than a naive reading of v2 suggested: radial and nonradial stability of these solutions has been analyzed in Refs. [6, 7, 8], with stability established for substantial regions of parameter space; what remains open is a fully nonlinear treatment (OP9).

7 Confrontation with Existing Data

7.1 Gravitational waves

(a) Ringdown spectroscopy. Scalar quasinormal modes of topological stars have been computed in Ref. [5]: for single-photon-sphere configurations the spectrum closely resembles that of a black hole sharing the photon sphere, while configurations with an inner stable photon sphere exhibit long-lived cavity modes and black-hole-like modes with reduced imaginary parts. Coupled electromagnetic–gravitational perturbations of the reduced four-dimensional solution have been studied in Refs. [9, 10]. What remains open, and constitutes the concrete near-term program, is the systematic confrontation of these computed spectra—and their completion in the full gravitational sector of the five-dimensional geometry—with LIGO–Virgo–KAGRA ringdown measurements as a function of b (OP1). Existing echo searches (O1–O3, $\Delta t \sim 0.05$–1 s) constrain only $b - 1 \lesssim 10^{-3}$, per §6.6. **(b)** Nonzero tidal Love numbers (vs. $k_2 = 0$ for Kerr) constrain the population via inspiral phasing; deformability analyses exist in Ref. [6], and their translation into LVK bounds is part of OP1. **(c)** Magnetically charged components would produce -1PN dipole emission, tightly bounded; neutral binaries evade this.

7.2 Shadows: an existing exclusion

EHT shadow diameters of M87* and Sgr A* agree with Kerr at the $\sim$10–20% level against dynamical masses [15]; both a $\geq 33\%$ deficit (Class II) and shadow absence (Class I) are excluded. **M87* and Sgr A* are not solitons of this class, for any b.** The theory survives only for unimaged compact objects; each future horizon-scale image repeats the binary test.

7.3 Solitonic dark matter

Smoothness gives $R_y \geq 2r_B$: smooth solitons are microscopic unless the extra dimension is macroscopic. For TeV-scale R_y, $M \sim 10^{7-8}$ kg $\sim 10^{-23}\,M_\odot$: below all microlensing sensitivity (Subaru–HSC window bounds only $M \gtrsim 10^{-16} M_\odot$), collisionless, cold, and topologically stable—an observationally viable CDM candidate, with the relic abundance open (OP3). The pseudo-isothermal profile of v1 is demoted to an empirical fit.

7.4 Colliders

$M_{KK} = \hbar c/R_y$; LHC resonance searches bound $M_{KK} \gtrsim \mathcal{O}(5\text{–}10)$ TeV, i.e. $R_y \lesssim 10^{-19}$ m, if Standard-Model gauge fields propagate in y as §2.2 requires. The viable corner is TeV-scale R_y, meshing with the $10^{-23} M_\odot$ dark-matter scale.

7.5 The size–hierarchy tension (honest disclosure)

$R_y \geq 2r_B$ (smoothness) versus $R_y \lesssim 10^{-19}$ m (collider) forces all smooth solitons to be subatomic. Astrophysical-mass mimickers require either $\mathbb{Z}_k$ orbifold quotients ($R_y \geq 2r_B/k$, $k \sim 10^{25}$ for km-scale r_B) or warped multi-scale internal geometry; neither is developed here (OP4). The theory bifurcates into a viable microscopic regime (dark matter, colliders) and an astrophysical regime requiring extension. Stating this explicitly is preferable to obscuring it.

7.6 Summary of falsifiable content

(1) Shadow deficit 33–43%—already excludes M87*, Sgr A*. (2) Ringdown spectra computed in the literature [5, 9, 10], awaiting systematic LVK confrontation (OP1); cavity spacing $\Delta f \simeq [11.7\, GM/c^3 \sqrt{(3/2-b)/(b-1)}]^{-1}$. (3) Separated echoes only for $b - 1 \lesssim 10^{-3}$. (4) Nonzero Love numbers. (5) KK resonances $\gtrsim$ TeV. (6) Solitonic CDM at $\sim 10^{-23} M_\odot$, relic abundance decisive.

8 Conclusion, Open Problems, and Outlook

8.1 Established results

(i) Gauge invariance as a theorem (§2.2); (ii) geometric parity violation with $\chi - 1 \propto \alpha/\beta$, CPT-consistent (§4.2–4.3); (iii) the torsion-decoupling lemma, embedding the topological-star literature intact within the unified theory; (iv) an analytic cavity-crossing formula consistent with published quasinormal-mode computations, plus the shadow-deficit window (4/7, 2/3); (v) non-empty falsified content (M87*, Sgr A*) and non-empty surviving content (microscopic solitonic dark matter).

8.2 Open problems

[C] computation; [T] construction; [F] foundational. **OP1** [C] Completion of the gravitational-sector QNM spectrum of the five-dimensional geometry and systematic confrontation with LVK ringdown data, building on Refs. [5, 9, 10, 6]. **OP2** [C] Echo delay beyond eikonal (wave transfer function). **OP3** [T] Relic abundance of $10^{-23} M_\odot$ solitons. **OP4** [T] Size–hierarchy resolution. **OP5** [T] Three chiral generations from an explicit $\mathcal{K}_7$ defect configuration. **OP6** [T] Moduli stabilization. **OP7** [T] $V - A$ from defect modes; EDM/atomic-parity residues of the torsion contact terms. **OP8** [C] $(33 - 2n_f)/12\pi$ from the moduli effective action. **OP9** [T] Fully nonlinear stability, extending Refs. [6, 7, 8]. **OP10** [F] Dynamical formation; microstate counting. **OP11** [F] UV completion. **OP12** [F] A convergent vacuum-energy computation.

8.3 Outlook

The v1→v2.1 revision makes a methodological point explicit: the original draft asserted a completed unification; the present draft delivers less and thereby claims more—every equation passes gauge-invariance and dimensional checks, every claim is either derived, cited to the literature, or flagged open, and the theory has been partially excluded by existing data (EHT) while remaining consistent with others (echo searches, QNM computations). The near-term program is OP1 and OP3. The conjecture linking weak-sector time-orientation to the macroscopic arrow of time is retained only as motivation: its naive form was incorrect (§4.3). We consider the honest cataloguing of such retractions part of the result.

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